Candidacy Exam
Department of Physics
August 26, 2006

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h $\hbar = h/2\pi$	$6.626 \times 10^{-34}\,\mathrm{J\,s}$ $1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales	$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$	

I–1. A particle of mass m moves in a plane under the influence of a central attractive force

$$F = -\frac{K}{r^2} e^{-\mu r},$$

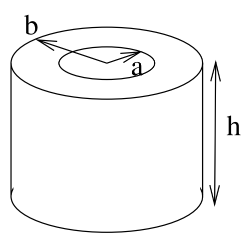
where K and μ are constants.

(a) Obtain an equation for the time dependence of r, the distance of the particle from the center of attraction.

(b) Determine the condition(s) on the constants such that circular motion of constant radius R is stable.

(c) Compute the frequency of small radial oscillations about this circular motion.

I–2. A hollow cylinder of inner radius a, outer radius b, and height h has a resistivity of ρ. What is the resistance $R = (V_b - V_a)/I$ for a current flowing from the outer to the inner radius.



I–3. The wave function of an electron in a one-dimensional infinite square well at time $t = 0$ is given by $\Psi(x, 0) = \sqrt{2/5}\psi_1(x) + \sqrt{3/5}\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are wave functions for the ground state and first excited stationary states of the system. [The well, of width a, extends from $x = 0$ to $x = a$, the energy eigenfunctions are $\psi_n(x) = \sqrt{2/a}\sin(n\pi x/a)$, and the eigenenergies are $E_n = n^2\pi^2\hbar^2/(2ma^2)$, where $n = 1, 2, 3, \ldots$.]

(a) Write down the wave function $\Psi(x, t)$ at time t in terms of $\psi_1(x)$ and $\psi_2(x)$.

(b) You measure the energy of an electron at time $t = 0$. Write down the possible values of the energy and the probability of measuring each.

(c) Calculate the expectation value of the energy in the state $\Psi(x, t)$ above.

I–4. Consider a collection of N independent particles, each bearing a magnetic moment μ which may be direction either up or down. Define n by saying that the number of spin-up moments is $(N+n)/2$, and the number of spin-down moments is $(N-n)/2$. The net magnetization is therefore $M = \mu n$. Assuming that there is no external magnetic field and that N is a very large number, derive the probability distribution $W(n)$ for having the magnetization $M = n\mu$. Derive the average value of the magentization, $\langle M \rangle$, and the average value of the squared magnetization, $\langle M^2 \rangle$.

Hint: Stirling's approximation may be useful: $\ln N! \simeq N \ln N - N$ for large N.

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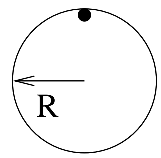
II–1. (a) State and prove the parallel axes theorem, which relates the moment of inertia of an object about an arbitrary axis to the moment of inertia about a parallel axis through the object's center of mass.

(b) A rigid loop of wire has radius R and is suspended from a frictionless pivot at its edge. Find the period of small oscillations under gravity. Neglect the radius of the wire and of the hook relative to R.



II–2. A magnetic field is generated by two coaxial plane coils, each of $n = 20$ turns of radius $a = 100\,\text{mm}$, separated by distance a.

(a) Calculate the magnetic field at a point midway between the coils when a current $I = 2\,\text{A}$ flows in each coil. The current in the two coils is parallel.

(b) Sketch the profile of the magnetic field strength summed along the axis of both coils.

II–3. (a) A spin-$\frac{1}{2}$ particle is in the presence of an external magnetic field applied in the z-direction, so that the relevant Hamiltonian is

$$H = -BS_z. \tag{II–1}$$

Which, if any, of the following are conserved quantities for this Hamiltonian: S^2, S_x, S_y, S_z?

(b) The normalized spin state of the particle at $t = 0$ is given by $|\Psi(t = 0)\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ represent spin states with, respectively, spin parallel to and opposite to the z-axis, respectively. The coefficients a and b are real numbers that are arbitrary except for the normalization constraint. Calculate the expectation values at time t of (i) S_z and (ii) S_x.

II–4. The equilibrium separation between hydrogen atoms in molecular H_2 is 0.08 nm, and the force constant of the bond is 580 $\mathrm{N m^{-1}}$.

(a) Estimate the minimum energy (in Joules) to cause each molecule to (i) rotate, (ii) vibrate.

(b) Roughly sketch the dependence on temperature of the specific heat capacity of hydrogen gas between 30 K and 1000 K.