Candidacy Exam
Department of Physics
January 14, 2006

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales	$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$	

I–1. A small mass m with initial velocity v goes by a star with a mass M at initial impact distance b. What is the distance of closest approach?

I–2. The space between the plates of thin parallel-plate capacitor is filled with a medium of conductivity σ and unit dielectric constant. The separation between the plates is d, and the plates are circular. A variable voltage $V = V_0 \sin \omega t$ is applied to the capacitor. Find the magnetic field inside the capacitor. Assume that the electric field between the plates is uniform, i.e., ignore edge effects. *In your calculation, assume that $\omega \ll c/L$ and $\omega \ll c^2\epsilon_0/(\sigma L^2)$, where L is maximum linear dimension of the capacitor.* What would affect your calculation if ω did not satisfy these conditions?

I–3. A paramagnetic solid is composed of spin-1/2 atoms (N per unit volume), each with a permanent magnetic dipole moment (μ per atom). In the presence of a magnetic field, of flux density B, the particles can occupy one of only two spin states, with the magnetic moments parallel or antiparallel to the B field, with energies $\pm\mu B$. The Boltzmann distribution tells us the number fN of dipoles oriented parallel and the number $(1-f)N$ antiparallel. In the whole problem, ignore dipole-dipole interactions.

(a) Use Boltzmann statistics to determine the fraction of moments that point parallel to the field at temperature T. What is the net dipole moment per unit volume (or magnetization) $M(T)$.

(b) The solid is held at a temperature of 1 K in a magnetic field of 1 T and is thermally isolated and is in equilibrium. Next the magnetic field is reduced to 0.3 T. If no dipoles change their orientation (i.e., no further thermal fluctuations have had an effect), to what temperature does the final distribution of dipoles correspond?

I–4. An electron is constrained to move around a circular ring of radius R. Its quantum mechanical wave function is therefore a function of the polar angle around the ring: $\psi(\theta)$. There is a uniform external classical magnetic field B perpendicular to the ring.

(a) First at $B = 0$, find the energy eigenstates and eigenfunctions. What is the ground state energy?

(b) Repeat with $B \neq 0$. (You will probably find it useful to obtain a vector potential for the magnetic field.) Show that the ground state is doubly degenerate when the magnetic flux through the ring is a half-integer multiple of ϕ_0. Here ϕ_0 is the elementary unit of magnetic flux, which is hc/e or h/e depending on the system of units.

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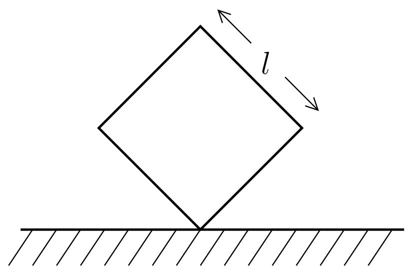
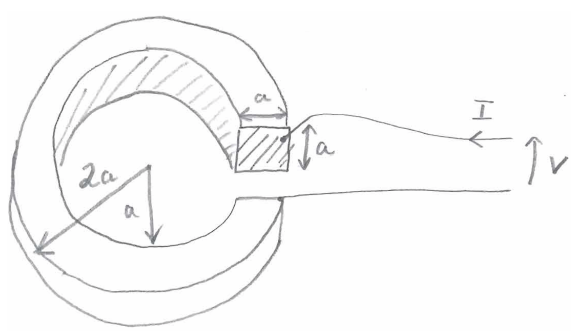
II–1. A homogeneous cube, each edge of which has a length l and whose mass is M, is initially in a position of unstable equilibrium with one edge in contact with a horizontal plane. The cube is then given a very small displacement and allowed to fall. Find the moment of inertia about a line that is through the center of mass and parallel to one of the edges of the cube. Then find the angular velocity of the cube when one face strikes the plane in the two cases: (i) the edge cannot slide on the plane, (ii) the edge can slide without friction.



II–2. Consider an object in the shape of a split ring. It is a circular ring with a square cross section, with a very small slit opened in it, and it is made of an electrically conducting material of resistivity ρ. The inner and outer radii of the ring are a and $2a$, and the thickness is a. Wires of negligible resistance are connected to the exposed faces.

Derive an expression, in terms of ρ and a, for the resistance R of the split ring as measured between the two wires.



II–3. (a) Define the term "chemical potential".

(b) What steps would you go through to calculate the chemical potential of a classical monatomic ideal gas, given its temperature, volume and the number of atoms? *There is no need to perform the calculation. Just explain clearly the steps needed to do the calculation.*

(c) In a container of such a gas is placed a solid on whose surface the atoms of the gas can be adsorbed. The adsorbed atoms form a two-dimensional ideal gas, with the energy of one atom being $\mathbf{p}^2/(2m) - \epsilon_0$, where $\mathbf{p}$ is its (two-component) momentum vector and ϵ_0 is the binding energy which holds the atom on the surface.

List the steps you would go through to obtain the number n' of atoms adsorbed per unit area of the solid's surface when the pressure of the surrounding gas is P and the temperature is T, and the system is in thermodynamic equilibrium.

II–4. The x-component of the spin of a spin 1 particle is measured, with the result $S_x = 0$. A second measurement is made, now of S_z. Find the possible values from the measurement and the corresponding probabilities.